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Research article

# When Bollinger meets Edgeworth: An application to the contrarian trading strategy

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## Abstract
The architecture of Bollinger Bands significantly contributes to informed decision-making in financial markets. This paper introduces two new techniques that incorporate Edgeworth correction for confidence intervals in classical Bollinger Bands (BB). In this regard, skewness and kurtosis are included to create Adjusted Bollinger Bands (ABB) with the aim of implementing a contrarian trading strategy. The performance of these techniques was evaluated on the 30 stocks of the Dow Jones Industrial Average Index and compared with performance metrics grouped by returns, risk, and risk-return measures, including the Sharpe ratio, Omega ratio, and Information ratio, and others. Results demonstrate the outperformance of our proposal based on ABB strategies over the classical BB for different periods, such as 6 and 10 days. The effectiveness of the approach presented in this paper exhibits a significant contrast to the traditional Bollinger Band methodology. Furthermore, these results are substantiated using robust statistical tests such as the Jonckheere-Terpstra Test.

**Keywords:** momentum; Edgeworth correction; confidence interval; Bollinger bands.

## Cuando Bollinger coincide con Edgeworth: una aplicación para la estrategia de trading contrario

## Resumen
La arquitectura de las Bandas de Bollinger contribuye significativamente a la toma de decisiones informadas en los mercados financieros. Este documento presenta dos nuevas técnicas que incorporan la corrección de Edgeworth para los intervalos de confianza en las Bandas de Bollinger clásicas (BB). En este sentido, se incluyen la asimetría y la curtosis para crear Bandas de Bollinger Ajustadas (BBA) con el objetivo de implementar una estrategia de trading contrario. El rendimiento de estas técnicas se evaluó en las 30 acciones del Índice del Promedio Industrial Dow Jones y se comparó con las métricas de rendimiento agrupadas por rendimientos, riesgo y medidas de riesgo-retorno, incluido el índice de Sharpe, el índice Omega y el índice de información, entre otros. Los resultados demuestran el rendimiento superior de nuestra propuesta basada en estrategias de BBA sobre el BB clásico durante diferentes períodos; por ejemplo, 6 y 10 días. La efectividad del enfoque presentado en este documento muestra un contraste significativo con la metodología tradicional de la Banda de Bollinger. Además, estos resultados se fundamentan mediante pruebas estadísticas robustas como la Prueba de Jonckheere-Terpstra.

**Palabras clave:** momentum; corrección de Edgeworth; intervalo de confianza; bandas de Bollinger.

## Quando Bollinger concorda com Edgeworth: uma aplicação para a estratégia de trading contrária

## Resumo
A arquitetura das Bandas de Bollinger contribui significativamente para a tomada de decisões informadas nos mercados financeiros. Este artigo apresenta duas novas técnicas que incorporam a correção de Edgeworth nos intervalos de confiança nas Bandas de Bollinger clássicas (BB). Nesse sentido, incluem-se a assimetria e a curtose para criar Bandas de Bollinger Ajustadas (BBA), com o objetivo de implementar uma estratégia de trading contrária. O desempenho dessas técnicas foi avaliado nas 30 ações do Índice Dow Jones Industrial Average e comparado a partir de métricas agregadas de rendimento, risco e medidas de risco-retorno, incluindo o índice de Sharpe, o índice Omega e o índice de informação, entre outros. Os resultados demonstram o desempenho superior da proposta baseada nas estratégias de BBA em relação às BB clássicas ao longo de diferentes horizontes temporais, como 6 e 10 dias. A eficácia da abordagem apresentada neste estudo evidencia um contraste significativo com a metodologia tradicional das Banda de Bollinger. Além disso, tais resultados são sustentados por testes estatísticos robustos, como o teste de Jonckheere-Terpstra.

**Palavras-chave:** momentum; correção de Edgeworth; intervalo de confiança; bandas de Bollinger.

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## 1. Introduction

In this paper, we propose Adjusted Bollinger Bands by using Edgeworth's expansions to adjust the confidence intervals used in the technical analysis and their implementation with the contrarian trading rule. To do that, we consider higher-order moments to incorporate the extensions into the confidence intervals that account for non-negligible characteristics of the underlying asset prices, such as skewness and kurtosis. As a result, we employ the Adjusted Bollinger Bands method as an alternative to the widely recognized Bollinger Bands technical analysis. The latter is a common way to predict price changes in markets, but its theory is debated. The Efficient Market Hypothesis (EMH), say that past prices shouldn't help predict future ones. Still, studies keep finding that simple strategies can make real money. One such strategy, Bollinger Bands, has recently attracted attention in the field. For example, Day et al. (2023), Vaidya (2021), and Ni et al. (2022) find that strategies using Bollinger bands can spot chances to profit in stocks and cryptocurrencies. Rahmadani and Patrisia (2025) show their application in developing countries too. This suggests that changes from the average and volatility bands might show market weaknesses or human errors that normal pricing models miss.

Studies also point out that financial returns often differ from Gaussian distribution, showing skewness, heavy tails, and high kurtosis. These factors can alter measures based on ideas of normal distribution. For example, Zhang et al. (2021) show that by taking into account the asymmetry of rapidly changing data, it is possible to predict how the market will change in the short term, thus enhancing the predictions of Gaussian models that only take volatility into account. Ahmed, Dutta, and Bouri (2022) also found that stock and commodity markets influence each other in terms of higher-order moments. Furthermore, Karagiorgis (2022) argues that skewness and kurtosis together can help us gain insight into non-normal patterns. In the field of derivatives, Chordia, Goyal, and Shanken (2021) state that risk-neutral skewness can provide us with information about informed trades and expected returns. Alexiou and Rompolis (2022) also make it clear that option moments can broaden our knowledge of stock returns.

Recent studies on technical trading support the idea that rule-based indicators can still predict results when non-Gaussian distributions are considered. Studies by Dockery and Todorov (2023) and Ergun, Molchanov, and Stork (2023) conclude that technical strategies work if downside risk, changing volatility, and nonlinear elements are considered. These results suggest that technical indicators' performance depends on the distribution of returns and that models considering asymmetry and kurtosis capture complex dynamics better than models considering only volatility.

In this paper, we propose adjusted Bollinger bands with Edgeworth-corrected confidence intervals implemented alongside the contrarian trading rule. Within the statistics literature, there is a growing focus on examining confidence intervals beyond the conventional approach. Bollinger Bands can be considered a particular instance of this approach. They focus more on moments than the first and second order. This research introduces a valuable improvement to technical trading. It challenges the common belief that Bollinger Bands are based on a Gaussian distribution. Our research proves that Bollinger Bands are merely a specific case of a broader class of distribution-aware confidence intervals. Additionally, we demonstrate that incorporating Edgeworth corrections, which account for skewness and kurtosis, yields more accurate trading signals in a contrarian system. Our detailed study of Dow Jones Industrial Average stocks included dominance tests, crisis-period evaluations, transaction-cost analysis, and Monte Carlo simulations that do not assume a Gaussian distribution.

This article highlights key findings and clarifies the statistical relationship between higher-order moments and the generation of technical signals. This relationship links asymptotic inference to the design of real-world trading systems. By presenting higher-order adjusted technical indicators as a viable alternative to traditional tools in contemporary financial markets, this article offers a valuable contribution. The study establishes a framework linking statistical theory, finance and trading methods. This framework will be of interest to academics and professionals, who are seeking alternatives to standard indicators in markets where assets exhibit non-Gaussian behavior.

The document is structured as follows: Section 2, Material and methods, describes the traditional Bollinger Bands and two confidence intervals built with Edgeworth expansions to construct the proposed Adjusted Bollinger Bands technique. The trading strategy with simulations is also presented. Section 3 shows the results of some performance indicators regarding return, risk, and risk-return with classical Bollinger Bands compared to our proposed Adjusted Bollinger Bands, adding robustness tests: crisis periods, transaction costs, and strategy implementation with a modified geometric Brownian motion with Cornish-Fisher expansion. Finally, Section 4 presents the conclusions.

## 2. Materials and Methods

This section presents an exhaustive analysis and the application of adjusted techniques with Edgeworth expansions to obtain the adjusted Bollinger Bands. In the first part, the traditional architecture of Bollinger Bands and its main applications in technical analysis—trend following, contrarian and squeeze frameworks—are reviewed. In the second part, two types of Adjusted Bollinger Bands confidence intervals with Edgeworth expansions are presented: the Edgeworth-corrected confidence intervals of the type studied by Hall (1983) and a Berry-Esseen for Edgeworth expansions (Hall & Jing, 1995) in order to construct the Adjusted Bollinger Bands.

### *2.1 Model 1: The Bollinger Bands*

Developed by John Bollinger in the 1980s, the construction of Bollinger Bands begins with a 20-period simple moving average (middle band) as a measure of central tendency. Additionally, bands are formed above and below this moving average, representing a 95% confidence interval delineated by a measure of volatility. Typically, these upper and lower bands are positioned two times away from the middle band based on the underlying price and an *n*-day standard deviation (Bollinger, 2002). The traditional Bollinger Band calculation procedure is as follows (Chen & Chuang, 2014). In the first step of the procedure, the *n*-day moving average at time *t* is calculated.

$$\mathrm{MA_t} = \frac{\sum_{i=0}^{n-1} \mathrm{P_{t-i}}}{\mathrm{n}}, \quad (1)$$

where $P_t$ is the price of the underlying asset at time *t*. In the second step, the *n*-day standard deviation is computed:

$$\mathrm{SD_t} = \left[\frac{\sum_{i=0}^{n-1} (\mathrm{P_{t-i}} - \mathrm{MA_t})^2}{(\mathrm{n}-1)}\right]^{0.5}. \quad (2)$$

Finally, the upper band (UB) and lower band (LB) for the confidence interval are assessed for each time *t*:

$$\mathrm{UB_t} = \mathrm{MA_t} + \mathrm{z_a}{*}\mathrm{SD_t}, \quad (3)$$

$$\mathrm{LB_t} = \mathrm{MA_t} - \mathrm{z_a}{*}\mathrm{SD_t}, \quad (4)$$

where $z_a = F^{-1}(\alpha)$ is the -level quantile of a standard normal distribution. These trading bands are lines plotted above and below the price behavior in the form of an envelope and are aimed at determining the probability of breaking through the bands. The latter serves as the basis for the decision-making process of buying or selling the analyzed assets. Therefore, it is expected that approximately 95% of the price activity occurs between the upper and lower Bollinger Bands, represented in the following confidence interval notation:

$$\mathrm{I_{2,a}} = \left(\overline{\mathrm{X}} - \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\mathrm{z_\beta}, \overline{\mathrm{X}} + \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\mathrm{z_\beta}\right) \quad (5)$$

where $\beta = \frac{(1+\alpha)}{2}$, $\overline{\mathrm{X}}$ represents the MA and $\hat{\sigma}$ stands for the SD given in equations (1) and (2), respectively. Treleaven & Hand (2013) identify opposing views in interpreting the price behavior associated with the Bollinger Bands, the so-called trend-following and contrarian frameworks.

Firstly, the trend-following viewpoint assumes that prices will continue to move in the direction of penetration. In other words, the penetration of the upper band suggests that prices will continue to move higher, indicating a buy condition. Likewise, the penetration of the lower band suggests that prices will continue to move lower, signaling a selling recommendation.

Secondly, the contrarian approach, the trading strategy applied in this study, asserts that a penetration of the upper or lower band is reflective of an over-reaction of the prices with a strong possibility of an impending trend reversal. The closer a security price is to its lower (upper) level, the more oversold it is, signaling a buy (sell) condition (Bollinger, 2002). The principle is that volatility tends to return to its mean, and after the price is located at the outer edge of a given band, it tends to trade back toward the median level, represented by the mid-line moving average. Another consideration concerns the significance of including the change in the Bollinger bandwidth to capture volatility, referred to as the "squeeze effect" (Bollinger, 2002). This implies that periods of low volatility are succeeded by periods of high volatility. A relatively narrow bandwidth (also known as the "squeeze") can anticipate a significant advance or decline in price.

Thirdly, the squeeze viewpoint posits that a new advance begins with a squeeze in the Bollinger bandwidth and a subsequent break above the upper band. Similarly, a new decline commences with a squeeze and a subsequent break below the lower band.

### *2.2 Model 2: Adjusted Bollinger Bands (ABB1)*

Our first proposal for the adjusted Bollinger Bands is based on the Edgeworth-corrected confidence intervals of the type examined by Pfanzagl (1979), Hall (1983), and Abramovich & Singh (1985). Let  denote the independent and identically distributed (i.i.d) random variables, in our case the asset prices, with finite third moment. In addition, $\overline{\mathrm{X}} = \mathrm{n}^{-1}\sum_{i\le n}\mathrm{X_i}, \hat{\sigma}^2 = \mathrm{n}^{-1}\sum_{i\le n}\left(\mathrm{X_i} - \overline{\mathrm{X}}\right)^2$ and $\hat{\gamma} = \hat{\sigma}^{-3}\mathrm{n}^{-1}\left(\mathrm{X_i} - \overline{\mathrm{X}}\right)^3$ are the estimators of $\mu = \mathrm{E}(\mathrm{X_i}), \sigma^2 = \mathrm{var}(\mathrm{X_i})$ and $\gamma = \frac{\mathrm{E}\left((\mathrm{X_i}-\mu)^3\right)}{\sigma^3}$, i.e., the mean, variance, and skewness, respectively. Accounting for skewness, Hall (1983) proposed the nominal -level one-sided interval, which is given by:

$$\mathrm{J_{1,a}} = \left(-\infty, \overline{\mathrm{X}} + \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\left\{\mathrm{z_a} - \mathrm{n}^{-\frac{1}{2}}\frac{1}{6}\hat{\gamma}\left(2\mathrm{z_a}^2+1\right)\right\}\right), \quad (6)$$

and,

$$\mathrm{J_{1,a}} = \left(\overline{\mathrm{X}} - \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\left\{\mathrm{z_a} - \mathrm{n}^{-\frac{1}{2}}\frac{1}{6}\hat{\gamma}\left(2\mathrm{z_a}^2+1\right)\right\}, \infty\right). \quad (7)$$

Whereas the two-sided interval is as follows:

$$\mathrm{J_{2,a}} = \left(\overline{\mathrm{X}} - \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\left\{\mathrm{z_\beta} + \mathrm{n}^{-\frac{1}{2}}\frac{1}{6}\hat{\gamma}\left(2\mathrm{z_\beta}^2+1\right)\right\}, \overline{\mathrm{X}} + \mathrm{n}^{-\frac{1}{2}}\hat{\sigma}\left\{\mathrm{z_\beta} - \mathrm{n}^{-\frac{1}{2}}\frac{1}{6}\hat{\gamma}\left(2\mathrm{z_\beta}^2+1\right)\right\}\right). \quad (8)$$

This confidence interval will be considered as an alternative to the usual confidence interval expressed in Equation (5) used in the Bollinger Bands construction, and it will be referred to as the ABB1 model. As observed, the Bollinger Bands is a special case of the ABB1 model when the skewness coefficient ($\hat{\gamma}$) is set at zero.

### *2.3 Model 3: Adjusted Bollinger Bands (ABB2)*

Subsequent to a study of Hall (1983), Hall & Jing (1995) proposed the Berry-Esseen theorem for Edgeworth expansions. This will be used in our work as a second alternative for the Bollinger Bands. Let μ = 0, and $T_0 = n^{\frac{1}{2}}\frac{\bar{X}}{\sigma}$, which represent the 'studentized mean'. If *X* has a non-singular distribution, then the one-term Edgeworth expansion is valid. That is,

$$P\left(T_0 \le x\right) = F\left(x\right) + n^{-\frac{1}{2}}\frac{1}{6}\gamma\left(2x^2+1\right)\phi\left(x\right) + o\left(n^{-\frac{1}{2}}\right), \quad (9)$$

Uniformly in *x*, as $n \to \infty$, where and denote the standard normal distribution and density functions, respectively, and $\gamma = E\left(X^3\right)var(X)^{-\frac{3}{2}}$ is the skewness of the sampling distribution (Chibishov, 1984; Hall, 1987). Then, Equation (8) may be extended to a longer expansion:

$$\begin{aligned} P\left(T_0 \le x\right) &= F\left(x\right) + n^{-\frac{1}{2}}\tfrac{1}{6}\gamma\left(2x^2+1\right)\phi\left(x\right) \\ &+ n^{-1}\{\tfrac{1}{12}ek\left(x^2-1\right) - \tfrac{1}{18}\gamma^2\left(x^4+2x^2-3\right) \\ &- \tfrac{1}{4}\left(x^2+3\right)\}\phi\left(x\right) + o\left(n^{-1}\right), \end{aligned} \quad (10)$$

Consistently in *x*, where *ek=E(X⁴)var(X)⁻²-3* denotes excess kurtosis. Therefore, a new confidence level can be constructed as:

$$\begin{aligned} P\left(T_0 \le x\right) &= F\left(x\right) + n^{-\frac{1}{2}}\tfrac{1}{6}\gamma\left(2x^2+1\right)\phi\left(x\right) \\ &+ n^{-1}\{\tfrac{1}{12}ek\left(x^2-1\right) - \tfrac{1}{18}\gamma^2\left(x^4+2x^2-3\right) \\ &- \tfrac{1}{4}\left(x^2+3\right)\}\phi\left(x\right) + o\left(n^{-1}\right), \end{aligned} \quad (11)$$

This new confidence interval considers not only skewness but also excess kurtosis and will be denominated the ABB2 model. As can be noted, the Bollinger Bands and the ABB1 model are special cases of the ABB2 model. Figure 1 illustrates the difference between models for the case of APPLE stock.

Next sections detail the trading strategy applied in our paper and present the results obtained using the three models presented above according to a battery of performance measures.

### *2.4 Trading Strategy*

The Contrarian trading rule (Bollinger, 2002) is tested with (1) the (classical) Bollinger Bands; (2) the ABB1 model, i.e., the confidence interval with Edgeworth expansion by Hall (1983), and (3) the ABB2 model, i.e., the confidence interval with the Berry-Esseen theorem for Edgeworth expansions proposed by Hall & Jing (1995). To achieve this, the 30 stocks that form the Dow Jones Industrial Average Index (Jeet & Vats, 2017) are employed between January 1, 2000, and December 31, 2022. We use the Bollinger Bands with 6-day, 10-day, 15-day, and 20-day moving average (MA) to generate automated trading signals 1, −1, and zero as follows. A buy signal (equals to 1) arises when the last price of a stock is at the lower line of the indicator, while the middle line is considered the resistance line. A sell signal (equals to −1) occurs when the last price is at the upper line of the indicator, while the middle line is considered the support level. Finally, the out-of-market includes situations where the signal is zero. We did not include any transaction and slippage costs to calculate performance measures, which might constitute an interesting refinement for further research. Figure 2 summarizes this strategy.

Several measures of trading performance, commonly utilized by experienced traders and academics (Balsara et al., 2009), are evaluated to test the success of the models with the contrarian trading strategy. A brief description of each of these measures, along with their mathematical descriptions can be found in the Appendix. Table 1 presents the performance measures grouped by return, risk, and risk-return indicators.

**Table 1.** Measures of trading performance.

| RETURN | RISK | RISK-RETURN |
|---|---|---|
| Cumulative Return | Annualized Standard Deviation | Sharpe Ratio |
| Annual Return | Maximum Drawdown (MDD) | Omega Ratio |
| | | Tracking Error |
| | | Information Ratio<br>Treynor Ratio |

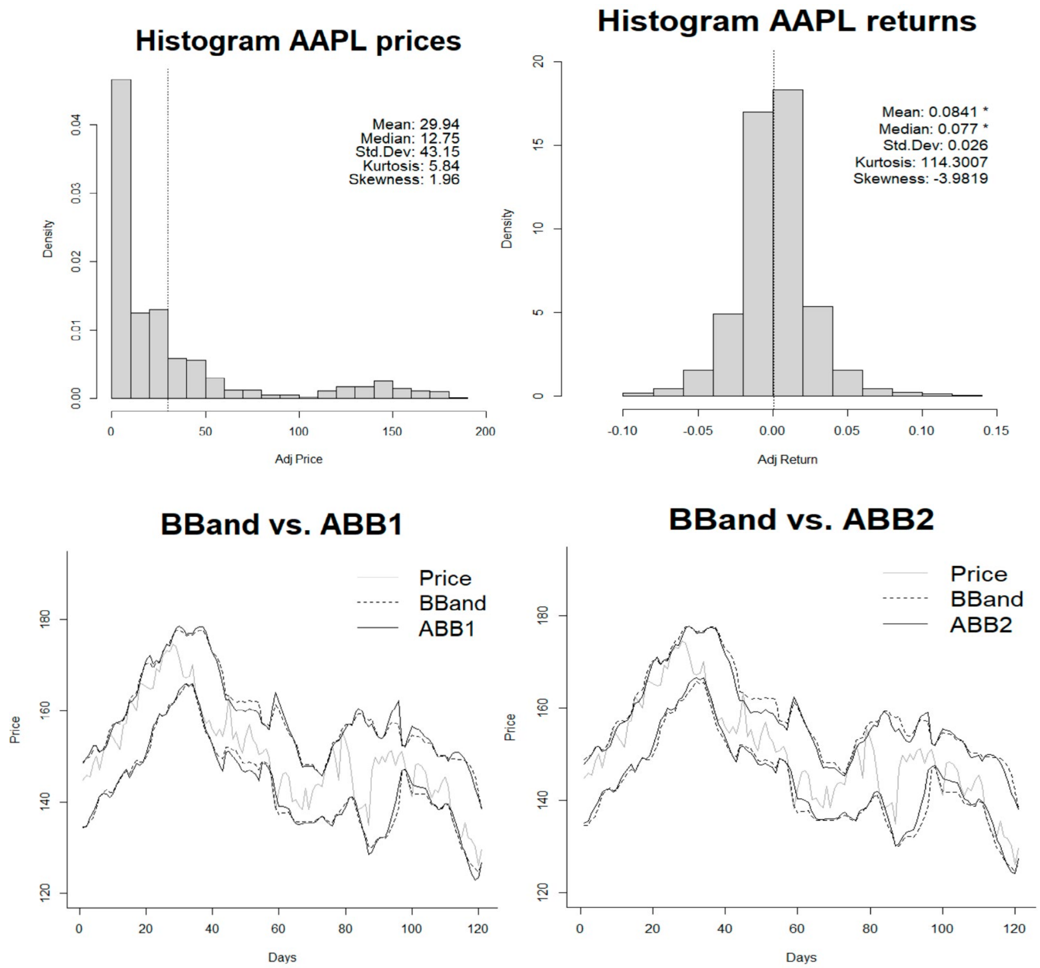
**Source:** own elaboration.

In addition, to statistically proof the validity of the best performance of our proposals on the median over the classical Bollinger bands in the different trading performance measures, we use the Non-Parametric Jonckheere Terpstra test ($T_{JT}$) (Ali et al., 2015) for ordered medians, where the null hypothesis is: H0: $n_1 = n_2$ with $n_i$ the population median for the ith population and the alternative hypothesis is that the population medians have an *a priori* ordering, e.g.: $H_A$: $n_1 \leqslant n_2 \leqslant \ldots \leqslant n_k$ ( or $n_1 \geqslant n_2 \geqslant \ldots \geqslant n_k$) with at least one of the inequalities being strict.

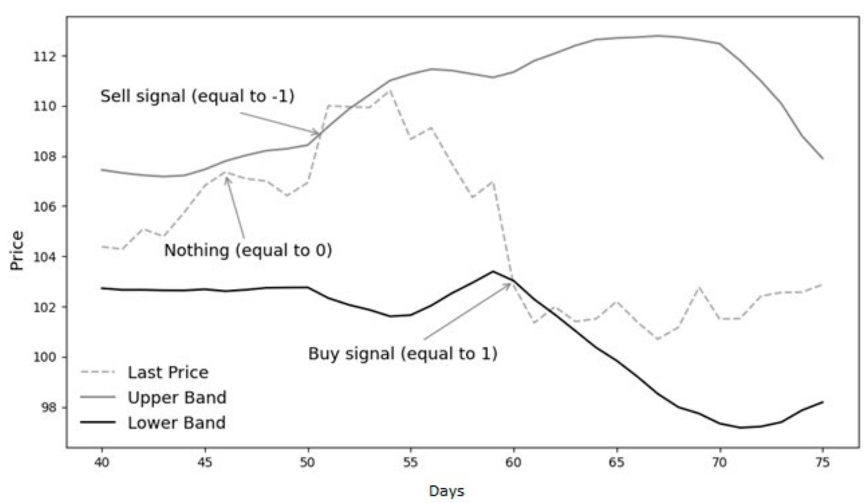
The statistic $T_{JT}$ is defined as $J = \sum U_{xy}$, where $U_{xy}$ is the number of observations in group greater than each observation in group . Therefore, the standardized $T_{JT}$ is computed as $Z = \frac{J - E(J)}{\sqrt{Var(J)}}$ with $E\left(J\right) = 0.25 * \left(N^2 - \sum_{j=1}^{k} n_j^{\,2}\right)$ and $Var\left(J\right) = N^2\left(2n+3\right) - \sum_{j=1}^{k} n_j^{\,2}\left(2n_j+3\right)$ where *j* is the number of the group, *N* is the total number of observations in all groups, $n_j$ is the observation in group j, and *k* is the total number of groups.

## 3. Model Results

Our findings clearly demonstrate that bands adjusted for higher-order moments outperform regular volatility-based bands in terms of both overall return and risk-adjusted return, particularly for short-term trading.




**Note:** AAPL stock price and returns (Upper Left and Right Panels) from January 1, 2008, to April 30, 2008. *N*=20. * Mean and Median multiplied by 100.
**Figure 1.** Difference between models for the case of APPLE stock.
**Source:** own elaboration.




**Note:** When the price is in an uptrend and touches the lower band, it is a buy signal (1). Meanwhile the price is in a downtrend and touches the upper band, it is a sell signal (-1), otherwise the signal is equal to zero.
**Figure 2.** Contrarian trading strategy explained.
**Source:** own elaboration.

### *3.1 Comparative Evaluation of Trading Strategies: Edgeworth vs. Bollinger Bands*

The universe of this study corresponds to the Dow Jones Industrial Average Index (DJIA) components (i.e., 30 stocks) for the analyzed period, which ranged between January 1, 2000, and December 31, 2022, on a daily frequency basis, for a total of 179,397 observations. Data was obtained from the Bloomberg Platform. Table 2 displays the descriptive statistics of the returns, calculated as , where is the price of the asset at time *t*.

**Table 2.** Descriptive Statistics for the analyzed stocks and the DJIA index.

| Stock | Mean | Std Deviation | Skewness | Excess kurtosis |
|---|---|---|---|---|
| IBM | 0.0041 | 0.0166 | -0.2837 | 8.4178 |
| AAPL | 0.0841 | 0.0260 | -3.9830 | 111.3980 |
| AXP | 0.0202 | 0.0228 | 0.1088 | 11.4298 |
| AMGN | 0.0247 | 0.0196 | 0.2407 | 6.1369 |
| BA | 0.0269 | 0.0224 | -0.4224 | 14.9123 |
| CAT | 0.0395 | 0.0204 | -0.1748 | 4.5122 |
| CSCO | -0.0022 | 0.0236 | 0.0607 | 9.6280 |
| CVX | 0.0252 | 0.0177 | -0.4420 | 19.9563 |
| DIS | 0.0187 | 0.0195 | -0.0450 | 9.6317 |
| HD | 0.0273 | 0.0195 | -1.1009 | 23.3084 |
| INTC | -0.0086 | 0.0237 | -0.5264 | 9.6980 |
| JNJ | 0.0232 | 0.0122 | -0.4927 | 14.3232 |
| JPM | 0.0175 | 0.0241 | 0.2239 | 13.9299 |
| KO | 0.0141 | 0.0133 | -0.1904 | 9.0681 |
| MCD | 0.0327 | 0.0147 | -0.1827 | 12.4120 |
| MMM | 0.0161 | 0.0151 | -0.2361 | 6.9338 |
| MSFT | 0.0244 | 0.0194 | -0.1675 | 9.2898 |
| NKE | 0.0513 | 0.0194 | -0.2455 | 11.3292 |
| PG | 0.0180 | 0.0137 | -3.3023 | 89.6203 |
| TRV | 0.0300 | 0.0183 | -0.2383 | 19.7552 |
| UNH | 0.0755 | 0.0197 | 0.1204 | 19.5338 |
| VZ | -0.0054 | 0.0152 | 0.1189 | 6.7707 |
| WBA | 0.0046 | 0.0182 | -0.3163 | 7.3637 |
| WMT | 0.0130 | 0.0150 | 0.0511 | 8.1617 |
| HON | 0.0238 | 0.0194 | -0.2287 | 13.5289 |
| MRK | 0.0094 | 0.0170 | -1.2866 | 26.9732 |
| T | -0.0113 | 0.0164 | -0.0371 | 7.1697 |
| GE | -0.0263 | 0.0212 | -0.0379 | 7.4038 |
| XOM | 0.0179 | 0.0168 | -0.0578 | 8.8756 |
| PFE | 0.0091 | 0.0160 | -0.1585 | 5.2446 |
| DJI | 0.0185 | 0.0119 | -0.3692 | 12.5198 |

**Note:** International Business Machine (IBM), Apple(AAPL), American Express (AXP), Amgen Inc. (AMGN), Boeing (BA), Caterpillar (CAT), American Telephone and Telegraph (T), Cisco (CSCO), Chevron (CVX), Disney (DIS), General Electric (GE), Exxon Mobil (XOM), Pfizer (PFE), Home Depot (HD), Walmart (WMT), Intel (INTC), Johnson & Johnson (JNJ), JP MORGAN (JPM), Coca Cola (KO), McDonald's (MCD), Minnesota Mining and Manufacturing (MMM), Merck (MRK), Microsoft (MSFT), Nike (NKE), Procter and Gamble (PG), Travelers (TRV), United Health (UNH), Verizon (VZ), Walgreens (WBA), Honeywell (HON), Dow Jones Industrial Average (DJI). The analyzed period ranged between January 1, 2000, and December 31, 2022, observed on a daily basis (179.397 observations).
**Source:** own elaboration.

As observed, Apple (AAPL) is the stock with highest daily means, excess kurtosis, standard deviation, and negative skewness. The only assets that present positive skewness are Amgen Inc (AMGN), JP Morgan (JPM), Walmart (WMT), Verizon (VZ), American Express (AXP), Cisco (CSCO), and United healthcare (UNH). All assets and the index present leptokurtic behavior and fat tails. All these features support the Non-Gaussianity property of daily return distributions (Cont, 2001).

For the analyzed period (January 1, 2000, to December 31, 2022), we calculated the performance indicators for return group: Cumulative Return and annual return; for risk group: Maximum Drawdown and annualized Standard Deviation; and for Risk-Return ratio group: Annualized Sharpe, Omega ratio, Tracking-Error, Information Ratio and Treynor Ratio (refer to the Appendix for additional indicator details). We measure the behavior of the Bollinger Bands and the Adjusted Bollinger Bands with Edgeworth expansions (ABB1 and ABB2) implemented with classical trading rule frameworks for 6, 10, 15 and 20 days. To summarize the results, the minimum, maximum and median of the performance measures were calculated for all the 30 assets comprising the Dow Jones Industrial Average Index. Individual asset results are available upon request.

The performance of the proposed model is compared with the behavior of the classical Bollinger Bands as the benchmark model. Tables 3, 4, 5, and 6 present the results for the performance measures with N=6 periods, N=10, N=15, and N=20 days, respectively, in the analyzed period between January 1, 2000, and December 31, 2022.

The medians of Cumulative Return, Annual Return, Sharpe, Tracking Error, and Information Ratio for the ABB1 and ABB2 models are higher than the median values of Cumulative Return, Annual Return, Sharpe, Tracking Error, and Information Ratio for the classical Bollinger Bands for N=6 and N=10.

Likewise, the median value of the Omega ratio for Edgeworth's ABB1 and ABB2 is higher than the Omega ratio of the classical Bollinger Bands for N=6. The median value of the Omega ratio for ABB2 exceeds the Omega ratio of classical Bollinger Bands for N=10. Similarly, the median values of Sharpe ratio and Tracking Error for ABB2 are higher than the Sharpe ratio and Tracking Error of the classical Bollinger Bands for N=15. Finally, the median values of Cumulative Return, Annual Return, and Information Ratio for the proposed model ABB2 are higher than those of the classical Bollinger Bands for N=15 and 20.

When applied to certain strategies, particularly over short periods (N = 6 and N=10), the Treynor ratio shows large negative numbers and considerable variation. This is primarily due to unreliable or close-to-zero beta calculations for individual assets, which can artificially inflate the ratio and reduce its interpretability. Therefore, although we include the Treynor ratio for thoroughness, Sharpe and Omega ratios provide a more reliable measure of performance, particularly given their robustness to tail risk and return asymmetry. These results are consistent with the current literature, which shows that strategies incorporating higher-order moments, particularly skewness and kurtosis, tend to perform better in the short term. Zhang, He, Zhang, and Wang (2021) provide one of the most compelling demonstrations of this phenomenon. Zhang,

**Table 3.** Performance of trading strategies on DJIA stocks using 6-day MA ('Full Sample').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1289 | -0.0060 | 0.0092 | 0.1066 | -0.3622 | 0.2601 | 0.1897 | -0.1841 | -148.8155 |
| **Max** | 0.1697 | 0.0069 | 0.1760 | 0.4767 | 0.4647 | 6.1743 | 0.1924 | -0.1158 | 6.9139 |
| **Mean** | -0.0041 | -0.0003 | 0.0693 | 0.2451 | -0.0124 | 1.3617 | 0.1904 | -0.1546 | -9.7708 |
| **Median** | -0.0220 | -0.0010 | 0.0653 | 0.2245 | -0.0674 | 0.8281 | 0.1902 | -0.1582 | -0.7683 |
| | | | | **ABB1** | | | | | |
| **Min** | 0.1418 | 0.0058 | 0.0765 | 0.5376 | 0.0905 | 1.1244 | 0.1930 | -0.1163 | -41.7996 |
| **Max** | 3.4894 | 0.0676 | 0.3053 | 1.5005 | 1.1070 | 2.6236 | 0.2166 | 0.1864 | 109.6205 |
| **Mean** | 1.3556 | 0.0347 | 0.1457 | 0.9319 | 0.5944 | 1.7522 | 0.2002 | 0.0274 | -0.5200 |
| **Median** | 1.2602 | 0.0362 | 0.1339 | 0.9478 | 0.6043 | 1.7286 | 0.1995 | 0.0355 | -4.7333 |
| **P-Value JT (BB-ABB1)** | 0.0002* | 0.0002* | 1 | 1 | 0.0002* | 0.0006* | 0.0002* | 0.0002* | 0.9370 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.8937 | 0.0282 | 0.0747 | 0.7542 | 0.4788 | 1.3885 | 0.1953 | -0.0046 | -16.3114 |
| **Max** | 14.0002 | 0.1252 | 0.2689 | 1.8752 | 1.3301 | 2.6439 | 0.2259 | 0.4254 | 76.6387 |
| **Mean** | 5.1044 | 0.0763 | 0.1397 | 1.2217 | 0.9935 | 2.1310 | 0.2076 | 0.2245 | 0.8858 |
| **Median** | 5.0115 | 0.0813 | 0.1282 | 1.1992 | 1.0158 | 2.1391 | 0.2068 | 0.2476 | -4.2991 |
| **P-Value JT (BB-ABB2)** | 0.0002* | 0.0002* | 1 | 1 | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.9038 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the period analyzed is January 1, 2000, and December 31, 2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 6. JT: Test Jonckheere-Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 4.** Performance of trading strategies on DJIA stocks using 10-day MA ('Full Sample').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0553 | -0.0025 | 0.0500 | 0.3754 | -0.0638 | 0.9530 | 0.1913 | -0.1635 | -29.6409 |
| **Max** | 1.0031 | 0.0307 | 0.3140 | 1.0205 | 0.6341 | 2.0246 | 0.2017 | 0.0081 | 57.5950 |
| **Mean** | 0.3494 | 0.0122 | 0.1501 | 0.7247 | 0.2684 | 1.4208 | 0.1960 | -0.0863 | 1.1531 |
| **Median** | 0.3595 | 0.0135 | 0.1532 | 0.7270 | 0.2721 | 1.4219 | 0.1959 | -0.0792 | -1.5703 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.0276 | -0.0012 | 0.0810 | 0.4976 | -0.0345 | 0.9842 | 0.1924 | -0.1570 | -27.8029 |
| **Max** | 1.4947 | 0.0406 | 0.3715 | 1.1215 | 0.7299 | 2.1084 | 0.2040 | 0.0576 | 17.2651 |
| **Mean** | 0.6776 | 0.0216 | 0.1616 | 0.8564 | 0.4021 | 1.5411 | 0.1985 | -0.0380 | -2.9061 |
| **Median** | 0.6476 | 0.0220 | 0.1451 | 0.8956 | 0.4411 | 1.5442 | 0.1983 | -0.0359 | -3.0584 |
| **P-Value JT (BB-ABB1)** | 0.0008* | 0.0004* | 0.7298 | 0.999 | 0.0102** | 0.0604 | 0.0014* | 0.0004* | 0.895 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.3062 | 0.0117 | 0.0693 | 0.6151 | 0.2079 | 1.2200 | 0.1929 | -0.0898 | -51.4973 |
| **Max** | 2.6485 | 0.0580 | 0.2550 | 1.4173 | 0.8689 | 2.3623 | 0.2153 | 0.1432 | 82.8284 |
| **Mean** | 1.3456 | 0.0361 | 0.1472 | 0.9849 | 0.5766 | 1.7402 | 0.2016 | 0.0335 | -2.1689 |
| **Median** | 1.2608 | 0.0362 | 0.1398 | 1.0172 | 0.6415 | 1.7924 | 0.2009 | 0.0356 | -3.8333 |
| **P-Value JT (BB-ABB2)** | 0.0002* | 0.0002* | 0.4974 | 1 | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.9788 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the period analyzed is January 1, 2000, and December 31 ,2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 10. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 5.** Performance of trading strategies on DJIA stocks using 15-day MA ('Full Sample').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0361 | -0.0016 | 0.0533 | 0.4267 | -0.0282 | 1.0005 | 0.1917 | -0.1537 | -60.5129 |
| **Max** | 1.1216 | 0.0333 | 0.3762 | 0.9519 | 0.7145 | 2.4032 | 0.2009 | 0.0211 | 32.9616 |
| **Mean** | 0.5166 | 0.0177 | 0.1244 | 0.6968 | 0.4069 | 1.6531 | 0.1959 | -0.0586 | -4.3269 |
| **Median** | 0.4906 | 0.0175 | 0.1136 | 0.7046 | 0.4272 | 1.6508 | 0.1960 | -0.0592 | -2.5649 |
| | | | | **ABB1** | | | | | |
| **Min** | 0.0401 | 0.0017 | 0.0513 | 0.4855 | 0.0307 | 1.0677 | 0.1926 | -0.1369 | -28.3994 |
| **Max** | 1.2305 | 0.0356 | 0.2594 | 1.0163 | 0.6985 | 2.4622 | 0.2047 | 0.0325 | 38.4392 |
| **Mean** | 0.5405 | 0.0182 | 0.1288 | 0.7408 | 0.3918 | 1.6000 | 0.1967 | -0.0558 | -3.3719 |
| **Median** | 0.4833 | 0.0173 | 0.1178 | 0.7439 | 0.3796 | 1.5625 | 0.1965 | -0.0600 | -2.5853 |
| **P-Value JT (BB-ABB1)** | 0.5384 | 0.5294 | 0.7202 | 0.8586 | 0.7010 | 0.8384 | 0.1994 | 0.5174 | 0.4900 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.1284 | 0.0053 | 0.0564 | 0.4880 | 0.0898 | 1.1222 | 0.1926 | -0.1181 | -22.3317 |
| **Max** | 2.0573 | 0.0499 | 0.2180 | 1.2151 | 0.7838 | 2.5953 | 0.2056 | 0.1017 | 24.8897 |
| **Mean** | 0.8095 | 0.0250 | 0.1200 | 0.7959 | 0.4976 | 1.7531 | 0.1979 | -0.0214 | -3.3152 |
| **Median** | 0.7253 | 0.0240 | 0.1092 | 0.7844 | 0.5182 | 1.7006 | 0.1972 | -0.0258 | -2.5309 |
| **P-Value JT (BB-ABB2)** | 0.006* | 0.0046* | 0.5040 | 0.9824 | 0.0194** | 0.1386 | 0.015** | 0.004* | 0.5806 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 200 and December 31, 2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 15. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively
**Source:** own elaboration.

**Table 6.** Performance of trading strategies on DJIA stocks using 20-day MA ('Full Sample').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0361 | -0.0016 | 0.0533 | 0.4267 | -0.0282 | 1.0005 | 0.1917 | -0.1537 | -60.5129 |
| **Max** | 1.1216 | 0.0333 | 0.3762 | 0.9519 | 0.7145 | 2.4032 | 0.2009 | 0.0211 | 32.9616 |
| **Mean** | 0.5166 | 0.0177 | 0.1244 | 0.6968 | 0.4069 | 1.6531 | 0.1959 | -0.0586 | -4.3269 |
| **Median** | 0.4906 | 0.0175 | 0.1136 | 0.7046 | 0.4272 | 1.6508 | 0.1960 | -0.0592 | -2.5649 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.0062 | -0.0003 | 0.0496 | 0.3895 | -0.0050 | 1.0316 | 0.1921 | -0.1489 | -23.3571 |
| **Max** | 1.7169 | 0.0445 | 0.2273 | 1.2777 | 0.7515 | 2.7331 | 0.2097 | 0.0755 | 3.5693 |
| **Mean** | 0.4493 | 0.0154 | 0.1190 | 0.7037 | 0.3495 | 1.6334 | 0.1964 | -0.0703 | -3.0737 |
| **Median** | 0.3988 | 0.0147 | 0.1046 | 0.6606 | 0.3421 | 1.5441 | 0.1946 | -0.0727 | -1.7125 |
| **P-Value JT (BB-ABB1)** | 0.6268 | 0.6342 | 0.7656 | 0.5756 | 0.7682 | 0.8034 | 0.4336 | 0.6072 | 0.4412 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.0990 | 0.0041 | 0.0640 | 0.4075 | 0.0830 | 1.1273 | 0.1923 | -0.1269 | -16.1402 |
| **Max** | 1.9280 | 0.0479 | 0.2614 | 1.2687 | 0.8500 | 3.1009 | 0.2093 | 0.0928 | 52.8112 |
| **Mean** | 0.5820 | 0.0193 | 0.1133 | 0.7368 | 0.4208 | 1.7553 | 0.1969 | -0.0504 | -1.4852 |
| **Median** | 0.5365 | 0.0189 | 0.0968 | 0.7013 | 0.4183 | 1.7026 | 0.1957 | -0.0516 | -1.9811 |
| **P-Value JT (BB-ABB2)** | 0.0476** | 0.0464** | 0.4836 | 0.8748 | 0.2032 | 0.3098 | 0.1304 | 0.0416** | 0.5260 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2000, and December 31,2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 20. JT: Test Jonckheere -Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

Feng, and Wei (2025) investigated the predictive ability of realized skewness in oil prices for exchange-rate movements in nine countries. They found that skewness improves forecasting accuracy only at short horizons, outperforming random-walk benchmarks and variance-based models. Similarly, Rehman, Sharif, and Ullah (2021) demonstrated that realized skewness and kurtosis drive weekly return predictability in equity markets. Their long–short portfolio strategy—buying stocks with high realized skewness and selling those with low skewness—generates statistically significant profits at the one-week horizon. However, this result disappears when longer windows are considered.

Studies on volatility forecasting and downside risk management further confirm the short-run value of higher-order moments. Rehman (2024) demonstrates that incorporating realized skewness and kurtosis into GARCH-type volatility models significantly improves the prediction of one-day and five-day volatility, though the benefits diminish at longer intervals. Similarly, Khademalomoom, Narayan, and Sharma (2019) demonstrate that GARCH models augmented with skewness and kurtosis provide more accurate intraday and short-term forecasts of exchange rate volatility. Notably, they demonstrate that trading strategies based on these higher-moment forecasts generate higher net profits and Sharpe ratios than strategies based on conventional GARCH models, even when transaction costs are considered.

Finally, the horizon-dependent nature of higher-moment relevance has been formalized in recent factor and risk-pricing research. Jin, Conlon, and Cotter (2023) analyze co-skewness and co-kurtosis across multiple return horizons. They document a pronounced horizon effect: higher-order co-moment risks are priced and economically relevant only over short horizons. Building on this, Gallo, Okhrin, and Storti (2024) found that models incorporating realized skewness and kurtosis significantly improved daily VaR and ES forecasts, especially when estimation windows were short and sensitive to tail dynamics. Together, these studies show that higher-order moment signals concentrate their predictive and strategic power in the short term when downside asymmetry and tail thickness dominate market dynamics.

Our results indicate that the confidence intervals based on the Edgeworth expansions generally outperform the classical Bollinger Bands in the classical trading strategy for the thirty stocks that form the Dow Jones Industrial Average Index in the analyzed period for short-term periods (N=6 and N=10). To confirm this, the following section presents the three models applied to the same assets during the specific periods comprising the two recent worldwide crises, namely, subprime and COVID-19 crises.

### *3.2 Robustness checks*

To prove the robustness of our model, we implement three different methodologies. First, we subject the proposed model to periods of crisis. Second, we implemented a robustness simulation that incorporates transaction costs to measure their impact on the proposed model (Balsara et al., 2009). Third, we use a Monte Carlo simulation of 30 synthetic assets exhibiting non-Gaussian behavior. We use a modified geometric Brownian motion with Cornish-Fisher to capture the behavior of the proposed strategies with higher-order moments.

#### *3.2.1 Crisis period*

As a robustness check, the performance indicators for return, risk, and risk-return groups were analyzed for the subprime crisis between January 1 and December 31, 2008; and the COVID-19 crisis in a period ranged between January 1 to December 31, 2020, both on a daily basis, for a total of 7,812 observations. Tables 7 to 14 present the results on the performance measures with *N*=6 10, 15 and 20 days.

For the 2008 and 2020 crises, it is observed that the median Cumulative Return and Annual Return for ABB2 are higher than the median Cumulative Return and Annual Return for the classical Bollinger Bands when N=6. Moreover, in the same periods of crisis (2008 and 2020),the median Tracking Error and Omega ratio for both the new techniques (ABB1 and ABB2 models) are higher than the median Tracking Error for the classical Bollinger Bands when *N*=6.

In the 2008 crisis, the medians of Tracking Error for ABB1 and ABB2 are higher than the median Tracking Error for the classical Bollinger Bands when N=10. For the 2020 crises, the median Cumulative Return and Annual Return for ABB1 are higher than the median Cumulative Return and Annual Return for the classical Bollinger Bands when N=6. Furthermore, the medians of Information Ratio for ABB1 and ABB2 are higher than the median Information Ratio for the classical Bollinger Bands when N=6. In the 2020 crisis, ABB2 demonstrates higher medians across Cumulative Return, Annual Return, Sharpe ratio, Omega ratio, Tracking Error, and Information Ratio compared to classical Bollinger Bands when N=10. Moreover, the median Tracking Error for ABB2 surpasses that of classical Bollinger Bands when N=15 during the same crisis period.

The findings for both crisis periods and full sample are in line with the simulation results, where the adjusted bands perform better for shorter periods of *N*, for the reasons explained in the Simulations section (2.5). Nevertheless, shorter periods for moving average calculations are employed in practice to capture short-term price fluctuations.

#### *3.2.2 Transaction costs*

Linear transaction costs (1%) were incorporated into the evaluation of the BB, HB, and HJB strategies to simulate realistic market friction. This mechanism works by detecting changes in the trading signal vector, where any non-zero difference identifies a trade execution event (entry, exit, or reversal). For every period where a trade is triggered, the strategy's gross return is penalized by applying a linear adjustment factor of , effectively reducing the net return on execution days to account for the cost of turnover.

As expected, the inclusion of linear transaction costs leads to an overall reduction in absolute performance across all strategies. However, and more importantly, the relative

**Table 7.** Performance of trading strategies on DJIA stocks using 6-day MA ('Subprime crisis period').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0651 | -0.0656 | 0.0000 | 0.0000 | -1.1940 | 0.0000 | 0.3778 | 0.8665 | -37.9130 |
| **Max** | 0.0734 | 0.0740 | 0.0651 | 0.9481 | 1.5037 | 36.5328 | 0.3858 | 1.2463 | 128.8565 |
| **Mean** | 0.0094 | 0.0095 | 0.0033 | 0.2115 | 0.4488 | 4.4864 | 0.3793 | 1.0711 | 11.3582 |
| **Median** | 0.0004 | 0.0004 | 0.0000 | 0.0764 | 1.0040 | 0.0000 | 0.3789 | 1.0478 | 7.1366 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.2441 | -0.2458 | 0.0006 | 0.2968 | -2.0449 | 0.0000 | 0.3708 | 0.3680 | -33.1620 |
| **Max** | 0.2702 | 0.2726 | 0.2802 | 3.3995 | 2.6552 | 243.2241 | 0.4310 | 1.5739 | 409.0257 |
| **Mean** | 0.0151 | 0.0153 | 0.0658 | 1.2745 | 0.2810 | 10.0953 | 0.3892 | 1.0584 | 13.1348 |
| **Median** | 0.0119 | 0.0120 | 0.0540 | 1.0731 | 0.2521 | 1.2564 | 0.3857 | 1.0659 | -1.9426 |
| **P-Value JT (BB-ABB1)** | 0.2400 | 0.2396 | 1 | 1 | 0.7886 | 0.0130** | 0.0002* | 0.5164 | 0.9500 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.2096 | -0.2111 | 0.0073 | 0.3862 | -1.6138 | 0.3526 | 0.3781 | 0.4505 | -66.3144 |
| **Max** | 0.4321 | 0.4362 | 0.2332 | 3.8420 | 3.0135 | 15.3873 | 0.4374 | 1.9046 | 15.0058 |
| **Mean** | 0.0829 | 0.0836 | 0.0680 | 1.6423 | 0.8659 | 2.8521 | 0.3969 | 1.2100 | -7.6829 |
| **Median** | 0.0391 | 0.0395 | 0.0504 | 1.4989 | 0.5406 | 1.4836 | 0.3933 | 1.1158 | -1.8607 |
| **P-Value JT (BB-ABB2)** | 0.0028* | 0.0024* | 1 | 1 | 0.1556 | 0.0072* | 0.0002* | 0.0506 | 0.9954 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2008, to December 31, 2008. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 6. JT: Test Jonckheere –Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 8.** Performance of trading strategies on DJIA stocks using 10-day MA ('Subprime crisis period').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1343 | -0.1353 | 0.0163 | 0.2638 | -2.0662 | 0.0000 | 0.3753 | 0.6752 | -36.3049 |
| **Max** | 0.2745 | 0.2769 | 0.1771 | 2.5235 | 1.8374 | 8.9392 | 0.4189 | 1.6291 | 107.1943 |
| **Mean** | 0.0081 | 0.0082 | 0.0541 | 1.0931 | -0.0143 | 1.6991 | 0.3858 | 1.0500 | 1.1246 |
| **Median** | 0.0084 | 0.0084 | 0.0425 | 0.8879 | 0.1628 | 1.1930 | 0.3829 | 1.0603 | 1.0795 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.1831 | -0.1845 | 0.0093 | 0.3421 | -2.0823 | 0.0122 | 0.3766 | 0.5415 | -30.5877 |
| **Max** | 0.3080 | 0.3108 | 0.2168 | 2.7302 | 2.5191 | 11.9271 | 0.4176 | 1.7049 | 46.0303 |
| **Mean** | 0.0110 | 0.0111 | 0.0694 | 1.3352 | 0.0078 | 2.2051 | 0.3907 | 1.0431 | -0.2404 |
| **Median** | 0.0190 | 0.0192 | 0.0589 | 1.1779 | 0.3151 | 1.3543 | 0.3879 | 1.0700 | -0.2932 |
| **P-Value JT (BB-ABB1)** | 0.5228 | 0.5200 | 0.8038 | 0.9776 | 0.4580 | 0.4826 | 0.0334* | 0.5156 | 0.3832 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.1692 | -0.1704 | 0.0093 | 0.4398 | -2.0187 | 0.0079 | 0.3764 | 0.5720 | -58.2772 |
| **Max** | 0.2427 | 0.2449 | 0.1806 | 2.7289 | 1.7218 | 5.8326 | 0.4195 | 1.5542 | 80.2897 |
| **Mean** | 0.0131 | 0.0132 | 0.0785 | 1.5113 | 0.0748 | 1.6418 | 0.3931 | 1.0424 | 1.4716 |
| **Median** | 0.0139 | 0.0141 | 0.0653 | 1.4541 | 0.2390 | 1.2459 | 0.3906 | 1.0651 | 1.6594 |
| **P-Value JT (BB-ABB2)** | 0.4622 | 0.4532 | 0.9756 | 0.9964 | 0.3400 | 0.3962 | 0.0050* | 0.5276 | 0.1562 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2008, to December 31, 2008. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* =10. JT: Test Jonckheere –Terpstra.* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 9.** Performance of trading strategies on DJIA stocks using 15-day MA ('Subprime crisis period').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| **BB** | | | | | | | | | |
| **Min** | -0.0819 | -0.0825 | 0.0000 | 0.2109 | -1.5107 | 0.0714 | 0.3775 | 0.7818 | -91.9146 |
| **Max** | 0.1096 | 0.1105 | 0.0861 | 2.2981 | 1.9078 | 12.6174 | 0.4103 | 1.3106 | 27.3610 |
| **Mean** | 0.0315 | 0.0317 | 0.0392 | 1.0160 | 0.5274 | 2.9031 | 0.3879 | 1.1053 | -2.4598 |
| **Median** | 0.0363 | 0.0366 | 0.0327 | 0.9794 | 0.6077 | 1.9743 | 0.3850 | 1.1219 | 1.2555 |
| **ABB1** | | | | | | | | | |
| **Min** | -0.1092 | -0.1100 | 0.0000 | 0.3432 | -1.5939 | 0.0482 | 0.3741 | 0.7303 | -251.0622 |
| **Max** | 0.1881 | 0.1897 | 0.2094 | 3.0739 | 1.9078 | 490.3076 | 0.4622 | 1.4451 | 25.2540 |
| **Mean** | 0.0245 | 0.0247 | 0.0495 | 1.1523 | 0.3848 | 19.9567 | 0.3911 | 1.0802 | -10.3112 |
| **Median** | 0.0131 | 0.0132 | 0.0327 | 1.1362 | 0.2526 | 1.2722 | 0.3866 | 1.0649 | 0.7284 |
| **P-Value JT (BB-ABB1)** | 0.7410 | 0.7430 | 0.7476 | 0.8068 | 0.7718 | 0.8081 | 0.2666 | 0.7662 | 0.6650 |
| **ABB2** | | | | | | | | | |
| **Min** | -0.0757 | -0.0763 | 0.0000 | 0.3432 | -0.9454 | 0.4011 | 0.3755 | 0.7936 | -251.0622 |
| **Max** | 0.1881 | 0.1897 | 0.2094 | 3.0759 | 2.2859 | 12.4323 | 0.4593 | 1.4945 | 44.1864 |
| **Mean** | 0.0409 | 0.0412 | 0.0507 | 1.2562 | 0.5540 | 2.3210 | 0.3924 | 1.1182 | -17.3363 |
| **Median** | 0.0353 | 0.0356 | 0.0365 | 1.1913 | 0.4919 | 1.6094 | 0.3895 | 1.1102 | -0.5004 |
| **P-Value JT (BB-ABB2)** | 0.4244 | 0.4106 | 0.8146 | 0.9738 | 0.5432 | 0.6560 | 0.0932 | 0.5012 | 0.8812 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2008, to December 31, 2008. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* =15. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 10.** Performance of trading strategies on DJIA stocks using 20-day MA ('Subprime crisis period').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| **BB** | | | | | | | | | |
| **Min** | -0.0967 | -0.0974 | 0.0000 | 0.2128 | -1.5422 | 0.0000 | 0.3784 | 0.7514 | -1070.1677 |
| **Max** | 0.1530 | 0.1544 | 0.1440 | 2.6319 | 2.1255 | 190.1128 | 0.3985 | 1.3856 | 117.8876 |
| **Mean** | 0.0282 | 0.0285 | 0.0351 | 0.9405 | 0.3676 | 13.2915 | 0.3863 | 1.1009 | -32.6263 |
| **Median** | 0.0118 | 0.0119 | 0.0258 | 0.9870 | 0.2810 | 1.3633 | 0.3854 | 1.0709 | 0.7183 |
| **ABB1** | | | | | | | | | |
| **Min** | -0.0967 | -0.0974 | 0.0000 | 0.2128 | -1.5422 | 0.0000 | 0.3784 | 0.7514 | -108.4493 |
| **Max** | 0.1663 | 0.1678 | 0.1440 | 2.6698 | 1.8374 | 19.8098 | 0.4012 | 1.4298 | 422.0984 |
| **Mean** | 0.0292 | 0.0295 | 0.0345 | 0.9354 | 0.4173 | 3.1973 | 0.3861 | 1.1041 | 15.6041 |
| **Median** | 0.0120 | 0.0121 | 0.0196 | 0.8496 | 0.4002 | 1.5447 | 0.3853 | 1.0766 | 0.7910 |
| **P-Value JT (BB-ABB1)** | 0.4522 | 0.4544 | 0.5240 | 0.4774 | 0.4192 | 0.4006 | 0.5588 | 0.4334 | 0.5852 |
| **ABB2** | | | | | | | | | |
| **Min** | -0.0967 | -0.0974 | 0.0000 | 0.2128 | -1.4281 | 0.0000 | 0.3760 | 0.7514 | -108.4493 |
| **Max** | 0.2279 | 0.2300 | 0.1573 | 2.6507 | 2.1186 | 108.8287 | 0.4057 | 1.5448 | 422.0984 |
| **Mean** | 0.0327 | 0.0330 | 0.0403 | 1.0335 | 0.3637 | 8.9091 | 0.3871 | 1.1095 | 15.9524 |
| **Median** | 0.0195 | 0.0196 | 0.0373 | 0.9838 | 0.4856 | 1.6652 | 0.3855 | 1.0897 | -0.4704 |
| **P-Value JT (BB-ABB2)** | 0.4866 | 0.4810 | 0.7762 | 0.7764 | 0.4844 | 0.4306 | 0.3894 | 0.4664 | 0.5624 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2008, to December 31, 2008. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* =20. JT: Test Jonckheere -Terpstra
* and ** significance at the 1% and 5% levels, respectively
**Source:** own elaboration.

**Table 11.** Performance of trading strategies on DJIA stocks using 6-day MA ('COVID-19 crisis effect').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0525 | -0.0529 | 0.0000 | 0.0000 | -1.2829 | 0.0000 | 0.3720 | -0.0873 | -118.9459 |
| **Max** | 0.0596 | 0.0601 | 0.0525 | 0.8364 | 1.4737 | 5.2735 | 0.3788 | 0.2125 | 90.7146 |
| **Mean** | -0.0016 | -0.0017 | 0.0094 | 0.2463 | 0.0698 | 0.7776 | 0.3733 | 0.0492 | -7.3343 |
| **Median** | 0.0009 | 0.0009 | 0.0041 | 0.1629 | 0.5058 | 0.0000 | 0.3725 | 0.0562 | -8.0321 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.0382 | -0.0385 | 0.0000 | 0.5515 | -0.7481 | 0.4816 | 0.3691 | -0.0489 | -49.1916 |
| **Max** | 0.2912 | 0.2938 | 0.0884 | 3.6461 | 2.7611 | 26.0501 | 0.5065 | 0.8091 | 252.3750 |
| **Mean** | 0.0681 | 0.0687 | 0.0408 | 1.2187 | 0.8334 | 2.9614 | 0.3855 | 0.2275 | 5.2168 |
| **Median** | 0.0480 | 0.0484 | 0.0346 | 1.0206 | 0.7144 | 1.8259 | 0.3802 | 0.1814 | -0.6204 |
| **P-Value JT (BB-ABB1)** | 0.0006* | 0.0006* | 1 | 1 | 0.0092* | 0.0002* | 0.0002* | 0.0008* | 0.2872 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.0675 | -0.0680 | 0.0016 | 0.6034 | -0.7114 | 0.5732 | 0.3737 | -0.1245 | -160.7665 |
| **Max** | 0.5120 | 0.5170 | 0.0874 | 4.7673 | 2.7194 | 73.5054 | 0.5526 | 1.1696 | 2785.2781 |
| **Mean** | 0.1498 | 0.1512 | 0.0456 | 1.8481 | 1.0593 | 5.8410 | 0.4049 | 0.4021 | 93.7282 |
| **Median** | 0.0974 | 0.0982 | 0.0442 | 1.3711 | 1.0505 | 2.4252 | 0.3889 | 0.3096 | -1.2412 |
| **P-Value JT (BB-ABB2)** | 0.0002* | 0.0002* | 1 | 1 | 0.0006* | 0.0002* | 0.0002* | 0.0002* | 0.1460 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2020, to December 31, 2020. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 6. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 12.** Performance of trading strategies on DJIA stocks using 10-day MA ('COVID-19 crisis effect').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1257 | -0.1266 | 0.0046 | 0.2143 | -1.5905 | 0.0000 | 0.3719 | -0.2508 | -53.1746 |
| **Max** | 0.2003 | 0.2021 | 0.1512 | 2.0315 | 1.7204 | 8.9506 | 0.4267 | 0.5662 | 17.4432 |
| **Mean** | 0.0026 | 0.0027 | 0.0485 | 0.9750 | -0.0004 | 1.7159 | 0.3825 | 0.0589 | -4.5172 |
| **Median** | 0.0050 | 0.0050 | 0.0411 | 0.9570 | 0.1295 | 1.1587 | 0.3762 | 0.0667 | -2.4180 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.0991 | -0.0999 | 0.0046 | 0.2143 | -1.3342 | 0.2759 | 0.3717 | -0.2063 | -107.7104 |
| **Max** | 0.1335 | 0.1347 | 0.1794 | 3.1556 | 1.4927 | 4.5134 | 0.4267 | 0.3999 | 27.8102 |
| **Mean** | 0.0029 | 0.0029 | 0.0565 | 1.1282 | 0.0607 | 1.3644 | 0.3840 | 0.0589 | -2.6673 |
| **Median** | 0.0024 | 0.0024 | 0.0456 | 1.0736 | 0.0875 | 1.0906 | 0.3789 | 0.0599 | -0.6702 |
| **P-Value JT (BB-ABB1)** | 0.4254 | 0.4128 | 0.7662 | 0.7740 | 0.4004 | 0.3988 | 0.2648 | 0.4178 | 0.1292 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.1419 | -0.1429 | 0.0073 | 0.30e05 | -1.1951 | 0.2683 | 0.3673 | -0.2778 | -16.0599 |
| **Max** | 0.5278 | 0.5330 | 0.1902 | 3.8947 | 2.1723 | 9.1458 | 0.4877 | 1.1337 | 409.6664 |
| **Mean** | 0.0648 | 0.0654 | 0.0520 | 1.4949 | 0.5913 | 2.5759 | 0.3954 | 0.2064 | 16.0846 |
| **Median** | 0.0366 | 0.0369 | 0.0428 | 1.4267 | 0.6228 | 1.8787 | 0.3883 | 0.1500 | -0.7093 |
| **P-Value JT (BB-ABB2)** | 0.0072* | 0.0084* | 0.7512 | 0.9978 | 0.0184** | 0.0238** | 0.0098* | 0.0060* | 0.0568 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2020, to December 31, 2020. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 10. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 13.** Performance of trading strategies on DJIA stocks using 15-day MA ('COVID-19 crisis effect').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1102 | -0.1111 | 0.0000 | 0.1853 | -1.6326 | 0.1437 | 0.3718 | -0.2139 | -45.6109 |
| **Max** | 0.0829 | 0.0836 | 0.1445 | 2.0189 | 2.1017 | 5.6862 | 0.4258 | 0.2750 | 128.8408 |
| **Mean** | 0.0000 | 0.0000 | 0.0346 | 0.6920 | 0.1132 | 1.4201 | 0.3776 | 0.0539 | 1.9155 |
| **Median** | 0.0040 | 0.0041 | 0.0296 | 0.6285 | 0.1263 | 1.1462 | 0.3739 | 0.0641 | -0.7009 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.1322 | -0.1332 | 0.0000 | 0.3287 | -1.6326 | 0.1437 | 0.3714 | -0.2658 | -9502.4682 |
| **Max** | 0.1075 | 0.1084 | 0.1484 | 2.0212 | 2.1502 | 6.8754 | 0.4257 | 0.3202 | 56.1200 |
| **Mean** | 0.0020 | 0.0020 | 0.0398 | 0.7941 | 0.1802 | 1.7293 | 0.3782 | 0.0590 | -322.5991 |
| **Median** | 0.0075 | 0.0076 | 0.0324 | 0.6853 | 0.1802 | 1.1536 | 0.3745 | 0.0736 | -1.7388 |
| **P-Value JT (BB-ABB1)** | 0.3704 | 0.3904 | 0.8102 | 0.9248 | 0.4076 | 0.4286 | 0.2138 | 0.3678 | 0.7110 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.1717 | -0.1730 | 0.0000 | 0.2622 | -1.6326 | 0.1437 | 0.3724 | -0.3593 | -9502.4682 |
| **Max** | 0.1552 | 0.1565 | 0.1748 | 2.0906 | 2.1502 | 20.4306 | 0.4257 | 0.4236 | 91.4900 |
| **Mean** | 0.0121 | 0.0122 | 0.0451 | 0.9404 | 0.2839 | 3.1104 | 0.3807 | 0.0846 | -311.1707 |
| **Median** | 0.0102 | 0.0103 | 0.0405 | 0.8635 | 0.2167 | 1.2158 | 0.3760 | 0.0812 | -2.6969 |
| **P-Value JT (BB-ABB2)** | 0.2398 | 0.2446 | 0.8816 | 0.9984 | 0.2632 | 0.3474 | 0.0134* | 0.2454 | 0.5258 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2020, to December 31, 2020. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 15. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 14.** Performance of trading strategies on DJIA stocks using 20-day MA ('COVID-19 crisis effect').

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1102 | -0.1111 | 0.0000 | 0.1853 | -1.6326 | 0.1437 | 0.3718 | -0.2139 | -45.6109 |
| **Max** | 0.0829 | 0.0836 | 0.1445 | 2.0189 | 2.1017 | 5.6862 | 0.4258 | 0.2750 | 128.8408 |
| **Mean** | 0.0000 | 0.0000 | 0.0346 | 0.6920 | 0.1132 | 1.4201 | 0.3776 | 0.0539 | 1.9155 |
| **Median** | 0.0040 | 0.0041 | 0.0296 | 0.6285 | 0.1263 | 1.1462 | 0.3739 | 0.0641 | -0.7009 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.1033 | -0.1041 | 0.0022 | 0.1995 | -1.8786 | 0.0000 | 0.3714 | -0.2229 | -145.7650 |
| **Max** | 0.1108 | 0.1117 | 0.1179 | 2.3151 | 2.3317 | 30.9674 | 0.4463 | 0.2951 | 95.2499 |
| **Mean** | -0.0081 | -0.0082 | 0.0483 | 0.8796 | -0.1963 | 2.2778 | 0.3817 | 0.0284 | -4.1171 |
| **Median** | -0.0103 | -0.0104 | 0.0354 | 0.7343 | -0.3110 | 0.7606 | 0.3760 | 0.0259 | -1.2730 |
| **P-Value JT (BB-ABB1)** | 0.6504 | 0.6466 | 0.7532 | 0.6964 | 0.6652 | 0.6360 | 0.3634 | 0.6512 | 0.2700 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.1001 | -0.1009 | 0.0022 | 0.3026 | -1.7665 | 0.0000 | 0.3714 | -0.2139 | -1872.8702 |
| **Max** | 0.1455 | 0.1467 | 0.1068 | 2.5212 | 2.3317 | 30.9674 | 0.4463 | 0.4282 | 95.2499 |
| **Mean** | 0.0072 | 0.0073 | 0.0453 | 0.9866 | -0.0214 | 2.7578 | 0.3841 | 0.0673 | -70.2302 |
| **Median** | -0.0152 | -0.0153 | 0.0414 | 0.9396 | -0.2668 | 0.7717 | 0.3769 | 0.0127 | -1.3778 |
| **P-Value JT (BB-ABB2)** | 0.4132 | 0.3980 | 0.7576 | 0.8764 | 0.4296 | 0.3808 | 0.0958 | 0.3908 | 0.3948 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2020, to December 31, 2020. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 20. JT: Test Jonckheere –Terpstra.
* and ** significance at the 1% and 5% levels, respectively
**Source:** own elaboration.

ranking of the models remains unchanged. Strategies based on adjusted Bollinger Bands (ABB1 and ABB2) continue to outperform classic Bollinger Bands in most risk-adjusted and return-adjusted performance measures, especially over short horizons (N = 6 and N = 10). This finding is fully consistent with the main results of the article and confirms the documented superior performance of the confidence intervals adjusted by the proposed methodologies. Tables 15 to 18 present the results on the performance measures with N=6 10, 15 and 20 days.

### *3.2.3 Simulations using a Cornish Fisher-Modified Geometric Brownian Motion*

To evaluate the performance of the proposed adaptive ABB1 for Hall's approach and ABB2 for Hall-Jing's approach, against classic BB, we used Monte Carlo simulations to generate price trajectories for 30 synthetic assets over a 500-day trading horizon. Unlike standard simulations that assume normality, the price trajectories followed a Geometric Brownian Motion (GBM) modified using the Cornish-Fisher expansion, which is described mathematically in the Appendix. The simulation parameters were set with a fixed random seed of 999 to ensure reproducibility and included an annual drift ranging from 0.05 to 0.15, volatility ranging from 0.25 to 0.45, skewness ranging from -0.8 to 0.3, and kurtosis ranging from 10 to 20. Strategies were back tested using different moving average window sizes of 6, 10, 15, and 20 days (N). Finally, to contrast the methodologies solidly, performance was evaluated using cumulative and annual returns, and risk-adjusted performance was evaluated using the Sharpe, omega, and information ratios, as well as tracking error. Tables 19 to 22 present the results of the performance measures with N = 6, 10, 15, and 20 days.

Results of the Cornish-Fisher simulation reveal a clear pattern dependent on the investment horizon. For short time horizons (N = 6 and N = 10), Cornish-Fisher-adjusted strategies outperformed classic Bollinger bands (BB) consistently in terms of cumulative and median annual returns, as well as risk-adjusted measures such as the Sharpe, Omega, and information ratios. This suggests that explicitly incorporating skewness and kurtosis through the Cornish-Fisher expansion can substantially improve the quality of signals in short-term contrarian trading, where non-normal return characteristics are most relevant. However, the efficacy of the adaptive methodologies diminishes as the moving average window expands, revealing a structural limitation in long-term applications.

As shown in Table 21 for N=15, the performance difference is reduced or even reversed and the classic BB has a better average Sharpe ratio (-0.1786) compared to ABB1 (-0.2964) and ABB2 (-0.2511). Regarding the Omega ratio classic BB (0.8565) versus ABB1 (0.7681) and ABB2 (0.8159), we can observe the same phenomenon in Table 22. with the same performance parameters. This phenomenon aligns with the theoretical expectation that as sample size N increases, the distribution of the moving average tends toward normality due to the Central Limit Theorem, rendering the specific adjustments for skewness and kurtosis less significant. Consequently, in longer periods such as N=15, while ABB2 shows isolated spikes in metrics like the Omega ratio, the consistency of the strategy becomes erratic due to the exclusion of recent relevant price changes. Therefore, the significance of high-order moments in trading bands is inversely proportional to the period length, positioning ABB1 and ABB2 as specialized tools for short-term, high-frequency trading regimes rather than long-term trend following.

## 4. Conclusions

In this paper, we compared three approaches for investment analysis in financial markets: Bollinger Bands (BB) and adjusted bands using Edgeworth expansions methodologies (Hall, 1983; Hall & Jing, 1995), named ABB1 and ABB2, applied to the contrarian trading strategy for the thirty stocks of the Dow Jones Industrial Average Index. The performance of these methodologies is assessed by three groups of indicators, including return, risk, and risk-return measures over a 20-year period. Moreover, the models' performance is tested in two crisis periods to verify the results obtained for the complete sample. Overall, the results show that confidence intervals with Edgeworth expansions for ABB1 and ABB2 using the contrarian trading strategy provide better median performance for indicators such as Cumulative Return, Annual return, Sharpe Ratio, Tracking error, Omega, and Information ratio (when N=6 and 10 days) than traditional Bollinger Bands (BB) for the entire sample period. We support the statistical overperformance of our proposed strategy based on ABB1 and ABB2 over the BB using the Jonckheere -Terpstra Test.

This research adds to the growing number of studies that question the assumption that many standard technical indicators follow a Gaussian distribution. By adding Edgeworth-corrected confidence intervals to Bollinger Bands, this article offers a statistical improvement that incorporates skewness and kurtosis into the creation of trading strategies. This is important because Bollinger Bands can be considered confidence intervals around a local mean. Data show that ignoring higher-order moments leads to poor signal creation for decision making. Therefore, results support the idea that the qualities of higher-order moments are very useful for obtaining better trading results in short periods of time.

These results are also in line with the crisis periods studied in this paper. Although the causes of the subprime crisis and COVID-19 pandemic are different, the effects of both crises result in higher volatility levels than the full period (i.e., 20 years) studied in our work. In these high uncertainty scenarios, the median Cumulative return, Annual return, Omega, and Tracking Error with our proposal (for N=6 and 10 days) perform better. This study shows that even when trading involves costs, the suggested adjusted

**Table 15.** Performance of trading strategies on DJIA stocks using 6-day MA -'Full Sample'.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1264 | -0.0059 | 0.0091 | 0.1048 | -0.3621 | 0.2601 | 0.1897 | -0.183 | -148.925 |
| **Max** | 0.1662 | 0.0067 | 0.1727 | 0.4667 | 0.4647 | 6.1743 | 0.1923 | -0.116 | 6.9143 |
| **Mean** | -0.0044 | -0.0003 | 0.0679 | 0.2397 | -0.0135 | 1.3692 | 0.1904 | -0.1541 | -9.9399 |
| **Median** | -0.0251 | -0.0011 | 0.0639 | 0.2159 | -0.0754 | 0.8274 | 0.1901 | -0.1585 | -0.9499 |
| | | | | **ABB1** | | | | | |
| **Min** | 0.1399 | 0.0057 | 0.075 | 0.5270 | 0.0911 | 1.1244 | 0.1929 | -0.1164 | -41.8243 |
| **Max** | 3.3621 | 0.0663 | 0.2999 | 1.4700 | 1.0963 | 2.6037 | 0.2158 | 0.1809 | 109.5628 |
| **Mean** | 1.3463 | 0.0347 | 0.141 | 0.8796 | 0.6059 | 1.7696 | 0.1998 | 0.0277 | -0.6204 |
| **Median** | 1.3026 | 0.037 | 0.1299 | 0.9112 | 0.6469 | 1.7908 | 0.1986 | 0.0397 | --4.8452 |
| **P-Value JT (BB-ABB1)** | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.9388 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.8616 | 0.0274 | 0.074 | 0.7461 | 0.471 | 1.3821 | 0.1952 | -0.0079 | -16.3092 |
| **Max** | 13.7597 | 0.1244 | 0.2665 | 1.8557 | 1.3321 | 2.7011 | 0.2253 | 0.4233 | 76.5519 |
| **Mean** | 5.1257 | 0.0766 | 0.1346 | 1.2128 | 1.0057 | 2.1536 | 0.2074 | 0.2269 | 1.3822 |
| **Median** | 5.0501 | 0.0816 | 0.1216 | 1.1906 | 1.0167 | 2.1791 | 0.2066 | 0.2581 | -4.0377 |
| **P-Value JT (BB-ABB2)** | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.0002* | 0.8858 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the period analyzed is January 1, 2000, and December 31, 2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 6. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 16.** Performance of trading strategies on DJIA stocks using 10-day MA - 'Full Sample'.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0721 | -0.0033 | 0.049 | 0.3667 | -0.0856 | 0.9305 | 0.1912 | -0.1672 | -33.1755 |
| **Max** | 0.8876 | 0.0281 | 0.3086 | 1.0017 | 0.6342 | 2.0232 | 0.2013 | 0.0049 | 57.6099 |
| **Mean** | 0.3245 | 0.0115 | 0.1484 | 0.7080 | 0.2581 | 1.4063 | 0.1958 | 0.0898 | 0.8662 |
| **Median** | 0.3324 | 0.0126 | 0.1524 | 0.7112 | 0.2754 | 1.3795 | 0.1955 | 0.0828 | -1.418 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.0411 | -0.0018 | 0.0794 | 0.4873 | -0.0528 | 0.9668 | 0.1923 | -0.1597 | -33.1447 |
| **Max** | 1.2888 | 0.0367 | 0.3653 | 1.0985 | 0.7392 | 2.1336 | 0.2035 | 0.0384 | 19.4615 |
| **Mean** | 0.622 | 0.0202 | 0.1594 | 0.8382 | 0.3861 | 1.522 | 0.1981 | -0.0444 | -3.0214 |
| **Median** | 0.5957 | 0.0206 | 0.1373 | 0.8763 | 0.4348 | 1.5415 | 0.1978 | -0.0432 | -3.0529 |
| **P-Value JT (BB-ABB1)** | 0.0008* | 0.0006* | 0.3000 | 0.0016* | 0.0118* | 0.0768 | 0.0036* | 0.0018* | 0.9060 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.2837 | 0.1009 | 0.0686 | 0.6096 | 0.1869 | 1.1985 | 0.1928 | -0.0932 | -67.2724 |
| **Max** | 2.6774 | 0.0584 | 0.2526 | 1.4033 | 0.8824 | 2.3623 | 0.2148 | 0.1456 | 55.056 |
| **Mean** | 1.2847 | 0.0349 | 0.1473 | 0.9731 | 0.5635 | 1.7256 | 0.2014 | 0.028 | -6.4393 |
| **Median** | 1.2052 | 0.035 | 0.143 | 1.0001 | 0.6356 | 1.7611 | 0.2009 | 0.0306 | -4.2929 |
| **P-Value JT (BB-ABB2)** | 0.0002* | 0.0002* | 0.4980 | 0.0002* | 0.0002* | 0.0004* | 0.0002* | 0.0002* | 0.9920 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the period analyzed is January 1, 2000, and December 31 ,2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 10.
* and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 17.** Performance of trading strategies on DJIA stocks using 15-day MA -'Full Sample'.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0498 | -0.0022 | 0.0523 | 0.4175 | -0.0401 | 0.9854 | 0.1917 | -0.1567 | -117.372 |
| **Max** | 1.154 | 0.034 | 0.37 | 0.9382 | 0.7146 | 2.492 | 0.2005 | 0.0251 | 32.9505 |
| **Mean** | 0.4982 | 0.0171 | 0.1234 | 0.6794 | 0.4024 | 1.6474 | 0.1956 | -0.0612 | -4.6328 |
| **Median** | 0.4654 | 0.0168 | 0.1137 | 0.6874 | 0.4434 | 1.6533 | 0.1957 | -0.0623 | --2.6131 |
| | | | | **ABB1** | | | | | |
| **Min** | 0.0238 | 0.001 | 0.0503 | 0.4762 | 0.0187 | 1.053 | 0.1925 | -0.1402 | -31.8825 |
| **Max** | 1.2624 | 0.0362 | 0.2547 | 0.9937 | 0.7244 | 2.5502 | 0.2041 | 0.0362 | 8.9113 |
| **Mean** | 0.5318 | 0.0179 | 0.1273 | 0.7255 | 0.3925 | 1.6057 | 0.1964 | -0.057 | -4.491 |
| **Median** | 0.4727 | 0.017 | 0.1193 | 0.7239 | 0.4002 | 1.5994 | 0.196 | -0.0611 | -2.5062 |
| **P-Value JT (BB-ABB1)** | 0.4334 | 0.4438 | 0.3190 | 0.1412 | 0.6600 | 0.8240 | 0.1726 | 0.4358 | 0.6430 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.1163 | 0.0048 | 0.0559 | 0.4826 | 0.101 | 1.1346 | 0.1926 | -0.1246 | -22.7592 |
| **Max** | 2.0252 | 0.0494 | 0.216 | 1.2033 | 0.7852 | 2.6019 | 0.2052 | 0.0999 | 24.8859 |
| **Mean** | 0.8133 | 0.0251 | 0.1187 | 0.7906 | 0.5032 | 1.7641 | 0.1978 | -0.0204 | -3.3004 |
| **Median** | 0.6963 | 0.0233 | 0.1075 | 0.7953 | 0.5131 | 1.7147 | 0.1975 | -0.0296 | -2.5195 |
| **P-Value JT (BB-ABB2)** | 0.0032* | 0.0018* | 0.5226 | 0.0088* | 0.0136* | 0.1282 | 0.0076* | 0.0022* | 0.6956 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 200 and December 31, 2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 15.
* and ** significance at the 1% and 5% levels, respectively
**Source:** own elaboration.

**Table 18.** Performance of trading strategies on DJIA stocks using 20-day MV -'Full Sample'.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.0045 | -0.0002 | 0.0428 | 0.3969 | -0.0036 | 1.0336 | 0.1911 | -0.1476 | -22.0606 |
| **Max** | 1.2254 | 0.0355 | 0.2231 | 1.2477 | 0.6360 | 2.4052 | 0.2086 | 0.0317 | 9.4587 |
| **Mean** | 0.4366 | 0.0153 | 0.1118 | 0.6794 | 0.3700 | 1.6887 | 0.1960 | -0.0704 | -3.3096 |
| **Median** | 0.3929 | 0.0145 | 0.0952 | 0.6397 | 0.3847 | 1.6674 | 0.1950 | -0.0747 | -1.6011 |
| | | | | **ABB1** | | | | | |
| **Min** | 0.0081 | 0.0004 | 0.0487 | 0.3810 | 0.0066 | 1.047 | 0.1920 | -0.1455 | -20.06 |
| **Max** | 1.6651 | 0.0436 | 0.2231 | 1.2525 | 0.7517 | 2.7331 | 0.2090 | 0.0719 | 3.2347 |
| **Mean** | 0.44 | 0.0151 | 0.117 | 0.6921 | 0.3491 | 1.6358 | 0.1962 | -0.0712 | -2.8871 |
| **Median** | 0.3976 | 0.0147 | 0.0996 | 0.6429 | 0.3334 | 1.5334 | 0.1947 | -0.0728 | -1.7137 |
| **P-Value JT (BB-ABB1)** | 0.6122 | 0.6120 | 0.2528 | 0.4156 | 07724 | 0.7794 | 0.4378 | 0.5926 | 0.4752 |
| | | | | **ABB2** | | | | | |
| **Min** | 0.0982 | 0.0041 | 0.0633 | 0.4032 | 0.0833 | 1.1273 | 0.1923 | -0.1267 | -11.8462 |
| **Max** | 1.8977 | 0.0474 | 0.2591 | 1.2557 | 0.8500 | 3.1009 | 0.2090 | 0.091 | 110.3203 |
| **Mean** | 0.5759 | 0.0191 | 0.1128 | 0.7334 | 0.4198 | 1.7583 | 0.1969 | -0.0507 | -1.0556 |
| **Median** | 0.5287 | 0.0187 | 0.0952 | 0.7191 | 0.4173 | 1.6920 | 0.1959 | -0.0515 | -1.8542 |
| **P-Value JT (BB-ABB2)** | 0.0402* | 0.0468* | 0.4402 | 0.1004 | 0.1912 | 0.3510 | 0.1078 | 0.040* | 0.5478 |

**Notes:** Performance statistics on the 30 Dow Jones' stocks when the analyzed period is January 1, 2000, and December 31,2022. ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 20. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 19.** Performance of trading strategies using 6-day moving averages.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.2682 | -0.1478 | 0.0034 | 0.0168 | -1.2241 | 0.0197 | 0.2090 | -0.2612 | -14.3259 |
| **Max** | 0.5628 | 0.2569 | 0.2964 | 0.5395 | 1.0694 | 36.4001 | 0.5785 | 0.8874 | 15.8914 |
| **Mean** | -0.0129 | -0.0109 | 0.1148 | 0.1069 | -0.2268 | 2.8668 | 0.2461 | 0.2671 | 0.0015 |
| **Median** | -0.0381 | -0.0197 | 0.0840 | 0.0890 | -0.3873 | 0.6590 | 0.2273 | 0.2853 | -1.0023 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.3139 | -0.1755 | 0.0148 | 0.0280 | -0.9799 | 0.2941 | 0.2138 | -0.2813 | --25.7509 |
| **Max** | 0.9230 | 0.3978 | 0.3699 | 0.2757 | 1.44428 | 19.1999 | 0.3510 | 1.3633 | 57.6658 |
| **Mean** | 0.0061 | -0.0034 | 0.1582 | 0.1498 | -0.0411 | 1.6358 | 0.2619 | 0.2875 | --0.6662 |
| **Median** | -0.0041 | -0.0021 | 0.1565 | 0.1492 | -0.0139 | 1.7551 | 0.2541 | 0.2964 | --0.6592 |
| **P-Value JT (BB-ABB1)** | 0.4626 | 0.4646 | 1 | 1 | 0.1820 | 0.1034 | 0.4378 | 0.4774 | 0.6236 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.4309 | -0.2508 | 0.0133 | 0.0315 | -0.9676 | 1.1783 | 0.2135 | -0.4861 | -8.6163 |
| **Max** | 0.7038 | 0.3138 | 0.4628 | 0.2728 | 1.2438 | 3.4783 | 0.3498 | 1.2194 | 568.9935 |
| **Mean** | 0.0469 | 0.0169 | 0.1749 | 0.1704 | 0.1557 | 1.3788 | 0.2748 | 0.3736 | 29.7318 |
| **Median** | 0.0470 | 0.0238 | 0.1574 | 0.1528 | 0.2226 | 1.2828 | 0.2576 | 0.4451 | 2.0070 |
| **P-Value JT (BB-ABB2)** | 0.0974 | 0.1010 | 1 | **1** | 0.0168* | 0.0394* | 0.0002* | 0.1062 | 0.0168* |

**Notes:** CBB: Classical Bollinger Bands, ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 6. JT: Test Jonckheere –Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 20.** Performance of trading strategies using 10-day moving averages.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.3773 | -0.2170 | 0.0107 | 0.0177 | -1.1409 | 0.0448 | 0.2195 | -0.3488 | -32.7937 |
| **Max** | 0.7800 | 0.3469 | 0.3958 | 0.2958 | 1.1728 | 23.1416 | 0.3877 | 1.1417 | 1152.6905 |
| **Mean** | -0.0336 | -0.0220 | 0.1243 | 0.1069 | -0.2359 | 1.7262 | 0.2446 | 0.3015 | 58.1551 |
| **Median** | -0.0300 | -0.0156 | 0.1090 | 0.0861 | -0.3523 | 0.6873 | 0.2301 | 0.3490 | 0.9908 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.3489 | -0.1987 | 0.0151 | 0.0131 | -1.2166 | 0.0863 | 0.2133 | -0.3531 | -12.5530 |
| **Max** | 1.3051 | 0.5392 | 0.3830 | 0.3765 | 1.4322 | 25.2186 | 0.4533 | 1.4005 | 127.2472 |
| **Mean** | -0.0012 | -0.0089 | 0.1330 | 0.1204 | -0.1292 | 1.9787 | 0.2512 | 0.3318 | 6.5535 |
| **Median** | -0.00008 | -0.0004 | 0.1200 | 0.1093 | -0.0030 | 1.0613 | 0.2382 | 0.3755 | 1.0802 |
| **P-Value JT (BB-ABB1)** | 0.3782 | 0.3544 | 0.4615 | 0.5687 | 0.2890 | 0.2452 | 0.2024 | 0.3958 | 0.4878 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.3548 | -0.2025 | 0.0154 | 0.0130 | -1.1883 | 0.1485 | 0.2133 | -0.3651 | -44.6305 |
| **Max** | 1.4198 | 0.5783 | 0.3567 | 0.3798 | 1.5225 | 23.3291 | 0.4555 | 1.4795 | 15.1119 |
| **Mean** | 0.0121 | -0.0021 | 0.1248 | 0.1156 | -0.1503 | 1.9678 | 0.2482 | 0.3507 | -3.2238 |
| **Median** | -0.0426 | -0.0222 | 0.1063 | 0.1000 | -0.3086 | 0.7647 | 0.2342 | 0.3155 | 0.4451 |
| **P-Value JT (BB-ABB2)** | 0.3444 | 0.3366 | 0.5312 | 0.5015 | 0.3308 | 0.1792 | 0.2370 | 0.3420 | 0.8868 |

**Notes:** CBB: Classical Bollinger Bands, ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 6. JT: Test Jonckheere –Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 21.** Performance of trading strategies using 15-day moving averages.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.2218 | -0.1226 | 0.0058 | 0.0131 | -1.1928 | 0.0566 | 0.2137 | -0.1767 | -48.6916 |
| **Max** | 0.3038 | 0.1484 | 0.2234 | 0.1803 | 1.2517 | 20.0709 | 0.2671 | 0.8906 | 31.8227 |
| **Mean** | -0.0064 | -0.0053 | 0.0803 | 0.0749 | -0.1242 | 2.4760 | 0.2282 | 0.3272 | -0.5560 |
| **Median** | -0.0201 | -0.0106 | 0.0503 | 0.0626 | -0.1786 | 0.8565 | 0.2231 | 0.3162 | 0.3635 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.2781 | -0.1563 | 0.0058 | 0.0187 | -1.2122 | 0.0410 | 0.2142 | -0.2559 | -79.9215 |
| **Max** | 0.2816 | 0.1382 | 0.2823 | 0.1857 | 1.4507 | 18.8869 | 0.2957 | 0.8678 | 39.5025 |
| **Mean** | -0.0171 | -0.0116 | 0.0977 | 0.0899 | -0.0914 | 2.4047 | 0.2341 | 0.3007 | -0.3634 |
| **Median** | -0.0230 | -0.0121 | 0.0676 | 0.0796 | -0.2964 | 0.7681 | 0.2255 | 0.3102 | 0.5807 |
| **P-Value JT (BB-ABB1)** | 0.6304 | 0.6144 | 0.4843 | 0.4576 | 0.4618 | 0.5320 | 0.1200 | 0.6246 | 0.4466 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.2781 | -0.1563 | 0.0121 | 0.0180 | -1.2444 | 0.0540 | 0.2141 | -0.2559 | -30.5543 |
| **Max** | 0.7197 | 0.3269 | 0.2823 | 0.2961 | 1.1684 | 42.3354 | 0.3598 | 1.1328 | 89.2780 |
| **Mean** | 0.0162 | 0.0035 | 0.1040 | 0.1061 | -0.1165 | 3.1645 | 0.2443 | 0.3310 | 3.9895 |
| **Median** | -0.0194 | -0.0101 | 0.0911 | 0.0961 | -0.2511 | 0.8159 | 0.2354 | 0.2920 | 0.4665 |
| **P-Value JT (BB-ABB2)** | 0.5812 | 0.5722 | 0.5789 | 0.5002 | 0.4676 | 0.4784 | 0.0138* | 0.6408 | 0.3788 |

**Notes:** Classical Bollinger Bands, ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). N = 15. JT: Test Jonckheere –Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration.

**Table 22.** Performance of trading strategies using 20-day moving averages.

| | Return | | Risk | | Risk-Return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Metric** | **Cum Return** | **Annual Return** | **Max Drawdown** | **Annualized Standard Deviation** | **Sharpe** | **Omega** | **Tracking Error** | **Information Ratio** | **Treynor** |
| | | | | **BB** | | | | | |
| **Min** | -0.1939 | -0.1074 | 0.0007 | 0.0084 | -1.1963 | 0.0465 | 0.2132 | -0.0556 | -185.1560 |
| **Max** | 0.4084 | 0.1979 | 0.2039 | 0.1771 | 1.3817 | 87.0006 | 0.2723 | 1.0844 | 8838.8115 |
| **Mean** | 0.0169 | 0.0067 | 0.0745 | 0.0754 | -0.0951 | 5.0288 | 0.2317 | 0.4280 | 299.2680 |
| **Median** | -0.0129 | -0.0068 | 0.0504 | 0.0652 | -0.1766 | 0.8294 | 0.2239 | 0.3957 | -0.2593 |
| | | | | **ABB1** | | | | | |
| **Min** | -0.1709 | -0.0941 | 0.0032 | 0.0084 | -1.1282 | 0.0599 | 0.2133 | 0.0009 | -279.2575 |
| **Max** | 0.4257 | 0.2056 | 0.2039 | 0.1773 | 1.3806 | 15.7845 | 0.2729 | 1.0999 | 27.6301 |
| **Mean** | 0.0207 | 0.0084 | 0.0718 | 0.0752 | -0.1043 | 2.6867 | 0.2317 | 0.4347 | -6.9717 |
| **Median** | -0.0179 | -0.0095 | 0.0481 | 0.0631 | -0.2750 | 0.6898 | 0.2238 | 0.3957 | 1.7520 |
| **P-Value JT (BB-ABB1)** | 0.5694 | 0.5824 | 0.4812 | 0.4516 | 0.6564 | 0.5758 | 0.4950 | 0.5392 | 0.1666 |
| | | | | **ABB2** | | | | | |
| **Min** | -0.1748 | -0.0963 | 0.0018 | 0.0084 | -1.1971 | 0.0465 | 0.2133 | -0.0086 | -64.5413 |
| **Max** | 1.0941 | 0.3269 | 0.1821 | 0.5470 | 1.3817 | 21.9742 | 0.5832 | 1.1105 | 44.4488 |
| **Mean** | 0.0432 | 0.0035 | 0.0744 | 0.0869 | -0.0845 | 3.2912 | 0.2418 | 0.4250 | 1.2541 |
| **Median** | -0.0301 | -0.0101 | 0.0533 | 0.0679 | -0.2750 | 0.6898 | 0.2235 | 0.3664 | 1.7438 |
| **P-Value JT (BB-ABB2)** | 0.6596 | 0.6588 | 0.7563 | 0.7412 | 0.5710 | 0.5292 | 0.5514 | 0.6218 | 0.1654 |

**Notes:** Classical Bollinger Bands, ABB1 is based on Hall (1983), and ABB2 on Hall & Jing (1995). *N* = 20. JT: Test Jonckheere –Terpstra. * and ** significance at the 1% and 5% levels, respectively.
**Source:** own elaboration

Bollinger Bands still do well. As expected, overall performance levels decline across all strategies when linear transaction costs are explicitly incorporated; however, the conclusions regarding the effectiveness of our proposed model remain valid insofar as, for short periods N=6 and N=10, the capture of higher-order moments based on the behavior of ABB1 and ABB2 outperforms the classic Bollinger bands.

Monte Carlo simulations, which use a revised geometric Brownian motion including Cornish-Fisher corrections, support the main points of this study: by adding skewness and kurtosis to the return process, the simulations copy the non-normal traits seen in real-world data. This lets us carefully test the suggested plans. The outcomes show that this effect changes with time. Cornish-Fisher-adjusted plans do better, both statistically and economically, in the short run, as shown by the Jonckheere-Terpstra test. But, as the trading period gets longer, this edge slowly fades. These results prove that changing for higher-order moments is most helpful for short-term contrarian trading. They also stress how vital it is to model  not normal return dynamics when judging technical trading rules. It should be noted that when transaction costs were implemented in the model proposed with different time windows, although the results in terms of profitability were eroded, the conclusions that support our work on the benefits of including higher-order moments in the strategy remained intact. Hence, they could be used in real life as an alternative to contemporary technical analysis.

Results provide traders and portfolio managers with practical advice for employing contrarian strategies in volatile markets. Adjusted Bollinger Bands (ABB) create trading signals that better reflect tail risk and asymmetry, resulting in higher total returns, Sharpe ratios, and Omega ratios, especially in short trades. These results remain consistent during crises and under transaction costs, suggesting that the ABB configuration is a valuable improvement for real-world use, not just a sample result. For users, this means that incorporating higher-order moment information can significantly improve risk-adjusted performance without compromising the simplicity and familiarity of the standard Bollinger Band structure.

The effectiveness of this approach, demonstrated through the utilization of Edgeworth expansions by Hall (1983) and Hall & Jing (1995), presents a compelling contrast with the traditional Bollinger Bands methodology. Future research could explore the implementation of this proposal, examining performance measures alongside various trading methodologies, including trend following and squeeze frameworks. Importantly, findings indicate that in shorter periods, these proposals adeptly capture the nuances of excess kurtosis and asymmetry. Hence, the recommendation is to consider their application in intraday trading schemes or even high-frequency trading. The main contribution of this paper is to incorporate confidence intervals that integrate skewness and kurtosis in the proposed contrarian trading strategy.

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## Appendix

Trade return (tr) is calculated using the return of the stock price and the previous day signal.

**Annual Return:** is the profit or loss on an investment over a one-year period.

$$\text{Annual Return} = [(1+\text{tr}_1)^*(1+\text{tr}_2)^*\ldots^*(1+\text{tr}_{m-1})^*(1+\text{tr}_n)]^{\frac{250}{T}} - 1 \quad \text{(A1)}$$

**Cumulative return:** is the aggregate amount that the investment has gained or lost over time, independent of the amount of time involved.

$$\text{Cumulative Return} = (1+\text{tr}_1)^*(1+\text{tr}_2)^*\ldots (1+\text{tr}_{n-1})^*(1+\text{tr}_n) \quad \text{(A2)}$$

$$\text{Annualized Standard Deviation}\sigma^A = \sqrt{252}^*\sigma \quad \text{(A3)}$$

**Maximum Drawdown** is an indicator of downside risk over a specified time period. It is the maximum observed loss from a peak to a trough of a portfolio before a new peak is attained.

$$\text{Maximum Drawdown} = \frac{(\text{Trough Value} - \text{Peak Value})}{\text{Peak Value}} \quad \text{(A4)}$$

**Omega** is an indicator that measures the likelihood of achieving a target return in comparison to the potential downside risk. It also considers the third and fourth momentum effect, i.e., skewness and kurtosis, which make it more useful compared to other indicators.

$$\Omega\left(\theta\right) = \frac{\int_{\theta}^{\infty}\left[1 - F\left(r\right)\right]dr}{\int_{-\infty}^{\theta}F\left(r\right)dr} \quad \text{(A5)}$$

**Sharpe Ratio** is defined as the difference between the returns of the investment and the risk-free return, divided by the standard deviation of the investment returns. It represents the additional amount of return that an investor receives per unit of increase in risk.

$$\text{Sharpe Ratio} = \frac{R_{TR} - R_f}{\sigma_{TR}} \quad \text{(A6)}$$

**Tracking-Error** is the divergence between the price behavior of a position or a portfolio and the price behavior of a benchmark. It is reported as a standard deviation percentage difference, which measures the difference between the return an investor receives and that of the benchmark they were attempting to imitate.

$$\text{Tracking Error} = \sigma_{TR}\sqrt{(1 - \rho_{TR,\text{RETURN DOW JONES}})^2} \quad \text{(A7)}$$

**Information Ratio** measures and compares the active return of an investment compared to a benchmark index relative to the volatility of the active return (also known as active risk or benchmark tracking risk).

$$\text{Information Ratio} = \frac{\text{Annualized excess return}}{\text{Annualized tracking error}} \quad \text{(A8)}$$

**Treynor Ratio is** a measurement of the returns earned in excess of that which could have been earned on an investment that has no diversifiable risk, per unit of market risk assumed.

$$\text{Treynor Ratio} = \frac{R_{TR} - R_f}{b_{TR}} \quad \text{(A9)}$$

**Cornish Fisher -Modified Geometric Brownian Motion**

To incorporate skewness and kurtosis, the standard normal innovation Z is replaced by a Cornish–Fisher adjusted quantile $Z_{CF}$:

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z_{CF}\right) \quad \text{(A10)}$$

where: $Z_{CF} = Z_P + \frac{\gamma_1}{6}\left(Z_P{}^2 - 1\right) + \frac{\gamma_2}{24}\left(Z_P{}^3 - 3Z_P\right) - \frac{\gamma_1{}^2}{36}\left(2Z_P{}^3 - 5Z_P\right)$, with: Z~N(0,1); $\gamma_1$ = skewness; and $\gamma_2$ = excess kurtosis.